▣ 수2 삼각함수 공식 총정리 ▣

삼각함수의 기초 개념

$\sin = \frac{\text{높이}}{\text{빗변}}, \cos = \frac{\text{거리}}{\text{빗변}}, \tan = \frac{\text{높이}}{\text{거리}}$

특수각의 삼각비

각 함수	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	없음

특수각의 제한적 확장

120° 150° 210° 330° −30° −120° 등

좌표와 삼각함수의 부호

① θ가 제1사분면의 각 : 모두 +

② θ가 제2사분면의 각 : sin(cosec)만 +

③ θ가 제3사분면의 각 : tan(cot)만 +

④ θ가 제4사분면의 각 : cos(sec)만 +

삼각함수의 변환공식

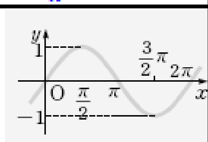
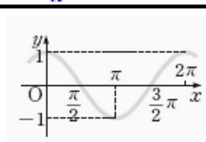
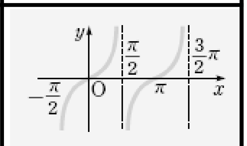
원래 삼각함수의 각 사분면에서의 부호 그대로 적용

삼각함수 $(90° \times n \boxed{\pm} \theta)$ ➲ (부호) 변환된 삼각함수θ

짝수일 때 : 그대로

홀수일 때 : $\begin{cases} \cos \leftrightarrow \sin \\ \tan \leftrightarrow \cot \end{cases}$

<삼각함수와 그래프>

	$y=\sin x$	$y=\cos x$	$y=\tan x$
그래프			
주 기	2π	2π	π
최 대	1	1	없다
최 소	-1	-1	없다

<삼각함수의 특성 공식>

$\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\sin^2\theta + \cos^2\theta = 1$

① 사인법칙

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$

② 사인 법칙의 변형

$\sin A = \frac{a}{2R}, \ \sin B = \frac{b}{2R}, \ \sin C = \frac{c}{2R}$

$a = 2R\sin A, b = 2R\sin B, c = 2R\sin C$

$\sin A : \sin B : \sin C = a : b : c$

	③ 제1코사인 법칙 $\begin{cases} a=b\cos C+c\cos B \\ b=c\cos A+a\cos C \\ c=a\cos B+b\cos A \end{cases}$ ④ 제2코사인 법칙 (두변의 길이와 한 각의 크기가 주어지는 경우) $\begin{cases} a^2=b^2+c^2-2bc\cos A \\ b^2=c^2+a^2-2ca\cos B \\ c^2=a^2+b^2-2ab\cos C \end{cases}$
<도형에의 활용>	① 부채꼴의 호의 길이 $l=r\theta$ ② 부채꼴의 넓이 $S=\frac{1}{2}r^2\theta=\frac{1}{2}rl$ ③ 두 변의 길이와 그 끼인각 이 주어지는 경우 $S=\frac{1}{2}ab\sin C=\frac{1}{2}bc\sin A=\frac{1}{2}ca\sin B$ ④ 외접원의 반지름 R이 주어진 경우 $S=\frac{abc}{4R}=2R^2\sin A\sin B\sin C$ ⑤ 내접원의 반지름 r이 주어진 경우 $S=sr$(단, $s=\frac{1}{2}(a+b+c)$) ⑥ 세변의 길이가 주어지는 경우(헤론의 공식) $S=\sqrt{s(s-a)(s-b)(s-c)}$(단, $s=\frac{1}{2}(a+b+c)$) ⑦ 두 변의 길이가 a, b인 평행사변형의 넓이 $S=ab\sin\theta$
삼각함수의 연산	15° 75° 275° -15° 등, sin ± sin, sin × cos 등
한 삼각함수 내 각도의 연산	① 덧셈정리 [1] $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$싸코코싸 [2] $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$ [3] $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$코코싸싸 [4] $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$ [5] $\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$ [6] $\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$ ② 2배각 공식 $\sin 2\theta=2\sin\theta\cos\theta$ $\cos 2\theta=\cos^2\theta-\sin^2\theta=2\cos^2\theta-1=1-2\sin^2\theta$ $\tan 2\theta=\frac{2\tan\theta}{1-\tan^2\theta}$ ③ 3배각 공식 $\sin 3\theta=3\sin\theta-4\sin^3\theta$, $\cos 3\theta=4\cos^3\theta-3\cos\theta$ ④ 반각 공식 $\sin^2\frac{\theta}{2}=\frac{1-\cos\theta}{2}$, $\cos^2\frac{\theta}{2}=\frac{1+\cos\theta}{2}$, $\tan^2\frac{\theta}{2}=\frac{1-\cos\theta}{1+\cos\theta}$

두 삼각함수의 연산	[1] 곱을 합차로 (1) $\sin\alpha\cos\beta=\frac{1}{2}\{\sin(\alpha+\beta)+\sin(\alpha-\beta)\}$ (2) $\cos\alpha\sin\beta=\frac{1}{2}\{\sin(\alpha+\beta)-\sin(\alpha-\beta)\}$ (3) $\cos\alpha\cos\beta=\frac{1}{2}\{\cos(\alpha+\beta)+\cos(\alpha-\beta)\}$ (4) $\sin\alpha\sin\beta=-\frac{1}{2}\{\cos(\alpha+\beta)-\cos(\alpha-\beta)\}$ [2] 합차를 곱으로 (1) $\sin A+\sin B=2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$ (2) $\sin A-\sin B=2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$ (3) $\cos A+\cos B=2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$ (4) $\cos A-\cos B=-2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
특수한 삼각함수의 연산	<삼각함수의 합성> [1] $a\sin\theta+b\cos\theta=\sqrt{a^2+b^2}\sin(\theta+\alpha)$ $\left(단, \cos\alpha=\frac{a}{\sqrt{a^2+b^2}}, \sin\alpha=\frac{b}{\sqrt{a^2+b^2}}\right)$ [2] $a\sin\theta+b\cos\theta=\sqrt{a^2+b^2}\cos(\theta-\beta)$ $\left(단, \cos\beta=\frac{b}{\sqrt{a^2+b^2}}, \sin\beta=\frac{a}{\sqrt{a^2+b^2}}\right)$ [3] 함수의 최댓값·최솟값 $a\sin\theta+b\cos\theta=\sqrt{a^2+b^2}\sin(\theta+\alpha)$에서 $-1\le\sin(\theta+\alpha)\le1$이므로 최댓값 : $\sqrt{a^2+b^2}$, 최솟값 : $-\sqrt{a^2+b^2}$
삼각방정식	[1] 각 삼각방정식의 특수해를 α, n을 임의의 정수라고 하면 ① $\sin x=a(\lvert a\rvert\le1)$의 일반해는 $x=n\pi+(-1)^n\alpha$ ② $\cos x=a(\lvert a\rvert\le1)$의 일반해는 $x=2n\pi\pm\alpha$ ③ $\tan x=a$의 일반해는 $x=n\pi+\alpha$ [2] 삼각방정식의 해법 ① 먼저 각을 통일 한다. ② 하나의 삼각함수로 고친 다음 이차방정식이나 치환에 의해 푼다. ③ $a\sin x+b\cos x=c$꼴로 유도, 삼각함수를 합성 ④ $\sin x=\sin\alpha$, $\cos x=\cos\alpha$, $\tan x=\tan\alpha$의 꼴로 유도, 일반해를 이용 ⑤ 합, 차를 곱으로 고치는 공식을 이용, $AB=0$꼴로 유도, $A=0$ 또는 $B=0$을 이용한다.

▣ 기본 수학 공식 ▣

0) 구의 체적 V = $\frac{4}{3}\pi r^3$,   구의 겉넓이 S = $4\pi r^2$

1) 지수법칙

(1) 지수의 확장

a을 실수 m, n을 양의 정수(자연수)하 할 때 다음과 같이 정의한다.

① $a^0 = 1$(단, a≠0)   ② $a^{-n} = \frac{1}{a^n}$(단, a≠0)

(2) 확장된 지수법칙

a〉 0, b〉 0이고 m, n을 유리수라 할 때

① $a^m \times a^n = a^{m+n}$ ② $a^m \div a^n = a^{m-n}$ ③ $(a^m)^n = a^{m \cdot n}$ ④ $(ab)^n = a^n b^n$ ⑤ $(\frac{b}{a})^n = \frac{b^n}{a^n}$

2) 로그의 정의

두수 x, y 사이에 $a^y = x$(a≠1, a0)인 관계가 있을 때 y는 a를 밑(base)으로 하는 x의 로그라하며

$y = \log_a x$

라고 나타낸다. 또 이때 x을 y의 진수라고 한다.

$a^y = x \leftrightarrow y = \log_a x$

3) 로그의 성질

(1) 로그의 기본공식

x〉 0, y〉 0, a≠1, a〉 0, n은 실수 일 때

① $\log_a a = 1$ ② $\log_a 1 = 0$ ③ $\log_a xy = \log_a x + \log_a y$

④ $\log_a \frac{x}{y} = \log_a x - \log_a y$ ⑤ $\log_a x^n = n \log_a x$

(2) 밑 변환공식

a≠1, a〉 0, b〉 0 일때

① $\log_a b = \frac{\log_e b}{\log_e a}$(e≠1, e〉 0)   ② $\log_a b = \frac{1}{\log_b a}$(b≠1)

(3) 이해하고 암기할 것

$\log 100 = 2$	$\log 10 = 1$	$\log 1 = 0$	$\log 0.1 = -1$
$\log 0.01 = -2$	$\log 2 = 0.3010$	$\log 3 = 0.4771$	

4) 행렬

(1) 행렬식

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\,a_{22}\,a_{33} + a_{12}\,a_{23}\,a_{31} + a_{13}\,a_{21}\,a_{32} - a_{11}\,a_{23}\,a_{32} - a_{12}\,a_{21}\,a_{33} - a_{13}\,a_{22}\,a_{31}$$

5) 적분 성질

(1) $\int cf(x)dx = c\int f(x)dx$ (단. c는 상수)   (2) $\int \{f(x)+g(x)\}dx = \int f(x)dx + \int g(x)dx$

(3) 이중적분(반복적분) $\int_0^1 \int_{-x}^{x} (1-x-y)dydx = \int_0^1 \left[y - xy - \frac{y^2}{2} \right]_{y=-x}^{y=x} dx = 2\int_0^1 \left(x - x^2\right)dx = 2\left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 = \frac{1}{3}$

▣ 미적분 공식 ▣

미분 사이트

Differentiation Formulas

$$\frac{d}{dx}k = 0 \tag{1}$$

$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x) \tag{2}$$

$$\frac{d}{dx}[k \cdot f(x)] = k \cdot f'(x) \tag{3}$$

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x) \tag{4}$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} \tag{5}$$

$$\frac{d}{dx}f(g(x)) = f'(g(x)) \cdot g'(x) \tag{6}$$

$$\frac{d}{dx}x^n = nx^{n-1} \tag{7}$$

$$\frac{d}{dx}\sin x = \cos x \tag{8}$$

$$\frac{d}{dx}\cos x = -\sin x \tag{9}$$

$$\frac{d}{dx}\tan x = \sec^2 x \tag{10}$$

$$\frac{d}{dx}\cot x = -\csc^2 x \tag{11}$$

$$\frac{d}{dx}\sec x = \sec x \tan x \tag{12}$$

$$\frac{d}{dx}\csc x = -\csc x \cot x \tag{13}$$

$$\frac{d}{dx}e^x = e^x \tag{14}$$

$$\frac{d}{dx}a^x = a^x \ln a \tag{15}$$

$$\frac{d}{dx}\ln|x| = \frac{1}{x} \tag{16}$$

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}} \tag{17}$$

$$\frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}} \tag{18}$$

$$\frac{d}{dx}\tan^{-1} x = \frac{1}{x^2+1} \tag{19}$$

$$\frac{d}{dx}\cot^{-1} x = \frac{-1}{x^2+1} \tag{20}$$

$$\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}} \tag{21}$$

$$\frac{d}{dx}\csc^{-1} x = \frac{-1}{|x|\sqrt{x^2-1}} \tag{22}$$

Integration Formulas

$$\int dx = x + C \tag{1}$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \tag{2}$$

$$\int \frac{dx}{x} = \ln|x| + C \tag{3}$$

$$\int e^x\,dx = e^x + C \tag{4}$$

$$\int a^x\,dx = \frac{1}{\ln a}a^x + C \tag{5}$$

$$\int \ln x\,dx = x\ln x - x + C \tag{6}$$

$$\int \sin x\,dx = -\cos x + C \tag{7}$$

$$\int \cos x\,dx = \sin x + C \tag{8}$$

$$\int \tan x\,dx = -\ln|\cos x| + C \tag{9}$$

$$\int \cot x\,dx = \ln|\sin x| + C \tag{10}$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C \tag{11}$$

$$\int \csc x\,dx = -\ln|\csc x + \cot x| + C \tag{12}$$

$$\int \sec^2 x\,dx = \tan x + C \tag{13}$$

$$\int \csc^2 x\,dx = -\cot x + C \tag{14}$$

$$\int \sec x \tan x\,dx = \sec x + C \tag{15}$$

$$\int \csc x \cot x\,dx = -\csc x + C \tag{16}$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C \tag{17}$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C \tag{18}$$

$$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{|x|}{a} + C \tag{19}$$

Differentiation

$(cu)' = cu' \qquad (c \text{ constant})$

$(u + v)' = u' + v'$

$(uv)' = u'v + uv'$

$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

$\frac{du}{dx} = \frac{du}{dy} \cdot \frac{dy}{dx} \qquad \text{(Chain rule)}$

$(x^n)' = nx^{n-1}$

$(e^x)' = e^x$

$(e^{ax})' = ae^{ax}$

$(a^x)' = a^x \ln a$

$(\sin x)' = \cos x$, $(\sin ax)' = a \cos x$

$(\cos x)' = -\sin x$, $(\cos ax)' = -a \sin x$

$(\tan x)' = \sec^2 x$

$(\cot x)' = -\csc^2 x$

$(\sinh x)' = \cosh x$

$(\cosh x)' = \sinh x$

$(\ln x)' = \frac{1}{x}$

$(\log_a x)' = \frac{\log_a e}{x}$

$(\arcsin x)' = \frac{1}{\sqrt{1 - x^2}}$

$(\arccos x)' = -\frac{1}{\sqrt{1 - x^2}}$

$(\arctan x)' = \frac{1}{1 + x^2}$

$(\text{arccot}\, x)' = -\frac{1}{1 + x^2}$

Integration

$\int uv'\, dx = uv - \int u'v\, dx$ (by parts)

$\int x^n\, dx = \frac{x^{n+1}}{n+1} + c \qquad (n \neq -1)$

$\int \frac{1}{x}\, dx = \ln |x| + c$

$\int e^{ax}\, dx = \frac{1}{a} e^{ax} + c$

$\int \sin x\, dx = -\cos x + c$

$\int \cos x\, dx = \sin x + c$

$\int \tan x\, dx = -\ln |\cos x| + c$

$\int \cot x\, dx = \ln |\sin x| + c$

$\int \sec x\, dx = \ln |\sec x + \tan x| + c$

$\int \csc x\, dx = \ln |\csc x - \cot x| + c$

$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \arctan \frac{x}{a} + c$

$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + c$

$\int \frac{dx}{\sqrt{x^2 + a^2}} = \text{arcsinh} \frac{x}{a} + c$

$\int \frac{dx}{\sqrt{x^2 - a^2}} = \text{arccosh} \frac{x}{a} + c$

$\int \sin^2 x\, dx = \frac{1}{2}x - \frac{1}{4} \sin 2x + c$

$\int \cos^2 x\, dx = \frac{1}{2}x + \frac{1}{4} \sin 2x + c$

$\int \tan^2 x\, dx = \tan x - x + c$

$\int \cot^2 x\, dx = -\cot x - x + c$

$\int \ln x\, dx = x \ln x - x + c$

$$\int e^{ax} \sin bx\, dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) + c$$

$$\int e^{ax} \cos bx\, dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx) + c$$

▣ 삼각함수 미분과 적분 ▣

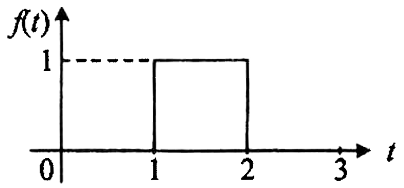
미분 <	삼각함수	> 적분
$\cos x$	sinx	$-\cos x$
$-\sin x$	cosx	$\sin x$
$\sec^2 x$	tanx	$-\ln\lvert\cos x\rvert$
$-\csc x\cot x$	cscx	$\ln\lvert\csc x-\cot x\rvert$
$\sec x\tan x$	secx	$\ln\lvert\sec x+\tan x\rvert$
$-\csc^2 x$	cotx	$\ln\lvert\sin x\rvert$
$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1}$x	$x\sin^{-1}x+\sqrt{1-x^2}$
$-\frac{1}{\sqrt{1-x^2}}$	$\cos^{-1}$x	$x\cos^{-1}x-\sqrt{1-x^2}$
$\frac{1}{1+x^2}$	$\tan^{-1}$x	$x\tan^{-1}x-\frac{1}{2}\ln(x^2+1)$
$-\frac{1}{x\sqrt{x^2-1}}$	$\csc^{-1}$x	$x\csc^{-1}x+\ln\lvert x+\sqrt{x^2-1}\rvert$
$\frac{1}{x\sqrt{x^2-1}}$	$\sec^{-1}$x	$x\sec^{-1}x-\ln\lvert x+\sqrt{x^2-1}\rvert$
$-\frac{1}{1+x^2}$	$\cot^{-1}$x	$x\cot^{-1}x+\frac{1}{2}\ln(x^2+1)$
$\cosh x$	sinhx	$\cosh x$
$\sinh x$	coshx	$\sinh x$
$\mathrm{sech}^2 x$	tanhx	$\ln(\cosh x)$
$-\mathrm{csch}\,x\coth x$	cschx	$\ln\lvert\tanh(\frac{x}{2})\rvert$
$-\mathrm{sech}\,x\tanh x$	sechx	$\tan(\sinh x)$
$-\mathrm{csch}^2 x$	cothx	$\ln\lvert\sinh x\rvert$
$\frac{1}{\sqrt{x^2+1}}$	$\sinh^{-1}$x	
$\frac{1}{\sqrt{x^2-1}}$	$\cosh^{-1}$x	
$\frac{1}{1-x^2}$	$\tanh^{-1}$x	
$-\frac{1}{x\sqrt{1+x^2}}$	csch^{-1}x	
$-\frac{1}{x\sqrt{1-x^2}}$	sech^{-1}x	
$\frac{1}{1-x^2}$	$\coth^{-1}$x	

합,곱셈,나눗셈의 미분규칙

- 상수배의 미분법 : $\frac{d}{dx}[cf(x)]=c\frac{d}{dx}f(x)$
- 합의 미분법 : $\frac{d}{dx}[f(x)+g(x)]=\frac{d}{dx}f(x)+\frac{d}{dx}g(x)$
- 곱의 미분법 (곱의 규칙,Product rule)
  - $\frac{d}{dx}[f(x)\,g(x)]=g(x)\,\frac{d}{dx}f(x)+f(x)\,\frac{d}{dx}g(x)$
  - $(uv)'=u'v+uv'$
- 몫의 미분법 (몫의 규칙,Quotient rule)
  - $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)=\frac{g(x)\,f'(x)+f(x)\,g'(x)}{(g(x))^2}$
  - $(\frac{u}{v})'=\frac{u'v+uv'}{v^2}$

▣ 라플라스 변환 ▣

f(t)	F(S)	f(t)	F(S)
$\delta(t)$	1	$\frac{t^n\ e^{-at}}{n!}$	$\frac{1}{(s+a)^{n+1}}$
$u(t)$	$\frac{1}{s}$	$\sin\ bt$	$\frac{b}{s^2+b^2}$
e^{-at}	$\frac{1}{s+a}$	$\cos\ bt$	$\frac{s}{s^2+b^2}$
t	$\frac{1}{s^2}$	$e^{-at}\ \sin\ bt$	$\frac{b}{(s+a)^2+b^2}$
$\frac{t^n}{n!}$	$\frac{1}{s^{n+1}}$	$e^{-at}\ \cos\ bt$	$\frac{s+a}{(s+a)^2+b^2}$
$t\ e^{-at}$	$\frac{1}{(s+a)^2}$		



1. 함수 식 f(t)=u(t-1)-u(t-2)
2. 라플라스 변환하면

$$F(S) = \frac{1}{s}e^{-s} - \frac{1}{s}e^{-s}$$

$$= \frac{1}{s}(e^{-s} - e^{-2s})$$

https://tutorial.math.lamar.edu/classes/de/Laplace_Table.aspx

Table Of Laplace Transforms

$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$	$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$
1. 1	$\frac{1}{s}$	2. $\mathbf{e}^{at}$	$\frac{1}{s-a}$
3. $t^n,\ n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4. $t^p, p>-1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5. $\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6. $t^{n-\frac{1}{2}},\ n=1,2,3,\ldots$	$\frac{1\cdot3\cdot5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7. $\sin(at)$	$\frac{a}{s^2+a^2}$	8. $\cos(at)$	$\frac{s}{s^2+a^2}$
9. $t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10. $t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11. $\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12. $\sin(at)+at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13. $\cos(at)-at\sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14. $\cos(at)+at\sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15. $\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16. $\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17. $\sinh(at)$	$\frac{a}{s^2-a^2}$	18. $\cosh(at)$	$\frac{s}{s^2-a^2}$
19. $\mathbf{e}^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20. $\mathbf{e}^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21. $\mathbf{e}^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22. $\mathbf{e}^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23. $t^n\mathbf{e}^{at},\ n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	24. $f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
25. $u_c(t)=u(t-c)$ Heaviside Function	$\frac{\mathbf{e}^{-cs}}{s}$	26. $\delta(t-c)$ Dirac Delta Function	$\mathbf{e}^{-cs}$
27. $u_c(t)f(t-c)$	$\mathbf{e}^{-cs}F(s)$	28. $u_c(t)g(t)$	$\mathbf{e}^{-cs}\mathfrak{L}\{g(t+c)\}$
29. $\mathbf{e}^{ct}f(t)$	$F(s-c)$	30. $t^n f(t),\ n=1,2,3,\ldots$	$(-1)^n F^{(n)}(s)$
31. $\frac{1}{t}f(t)$	$\int_s^\infty F(u)\,du$	32. $\int_0^t f(v)\,dv$	$\frac{F(s)}{s}$
33. $\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	34. $f(t+T)=f(t)$	$\frac{\int_0^T \mathbf{e}^{-st}f(t)\,dt}{1-\mathbf{e}^{-sT}}$
35. $f'(t)$	$sF(s)-f(0)$	36. $f''(t)$	$s^2F(s)-sf(0)-f'(0)$
37. $f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$		

▣ 라플라스와 Z 변환 ▣

Entry	Time Domain *(Note)*	Laplace Domain	Z Domain (t=kT)
unit impulse	$\delta(t)$ unit impulse	1	1
unit step	$u(t)$, $\gamma(t)$ *(Note)*	$\Gamma(s)=\frac{1}{s}$	$\frac{z}{z-1}$
unit step time delay	$u(t-\tau)$	$\frac{1}{s}e^{-\tau s}$	
ramp	t	$\frac{1}{s^2}$	$T\frac{z}{(z-1)^2}$
parabola	t^2	$\frac{2}{s^3}$	$T^2\frac{z(z+1)}{(z-1)^3}$
t^n (n is integer)	t^n	$\frac{n!}{s^{(n+1)}}$	
exponential	e^{-at}	$\frac{1}{s+a}$	$\frac{z}{z-e^{-aT}}$
power	b^k $(b=e^{-aT})$		$\frac{z}{z-b}$
time multiplied exponential	te^{-at}	$\frac{1}{(s+a)^2}$	$T\frac{ze^{-aT}}{(z-e^{-aT})^2}$
Asymptotic exponential	$\frac{1}{a}(1-e^{-at})$	$\frac{1}{s(s+a)}$	$\frac{z(1-e^{-aT})}{a(z-1)(z-e^{-aT})}$
double exponential	$\frac{e^{-at}-e^{-bt}}{(b-a)}$	$\frac{1}{(s+a)(s+b)}$	$\frac{z(e^{-aT}-e^{-bT})}{(b-a)(z-e^{-aT})(z-e^{-bT})}$
asymptotic double exponential	$\frac{1}{ab}\left(1-\frac{be^{-at}-ae^{-bt}}{(b-a)}\right)$	$\frac{1}{s(s+a)(s+b)}$	
asymptotic critically damped	$\frac{1}{a^2}(1-e^{-at}-ate^{-at})$	$\frac{1}{s(s+a)^2}$	$\frac{(1-e^{-Ta}(1+Ta))z^2+e^{-Ta}(Ta-1+e^{-Ta})z}{a^2(z-e^{-Ta})^2(z-1)}$
differentiated critically damped	$(1-at)e^{-at}$	$\frac{s}{(s+a)^2}$	$\frac{z(z-(Ta+1)e^{-Ta})}{(z-e^{-Ta})^2}$
sine	$\sin(\omega_0 t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\frac{z\sin(\omega_0 T)}{z^2-2z\cos(\omega_0 T)+1}$
cosine	$\cos(\omega_0 t)$	$\frac{s}{s^2+\omega_0^2}$	$\frac{z(z-\cos(\omega_0 T))}{z^2-2z\cos(\omega_0 T)+1}$
decaying sine	$e^{-at}\sin(\omega_d t)$	$\frac{\omega_d}{(s+a)^2+\omega_d^2}$	$\frac{ze^{-aT}\sin(\omega_d T)}{z^2-2ze^{-aT}\cos(\omega_d T)+e^{-2aT}}$
decaying cosine	$e^{-at}\cos(\omega_d t)$	$\frac{s+a}{(s+a)^2+\omega_d^2}$	$\frac{z(z-e^{-aT}\cos(\omega_d T))}{z^2-2ze^{-aT}\cos(\omega_d T)+e^{-2aT}}$

Entry	Time Domain *(Note)*	Laplace Domain	Z Domain (t=kT)
unit impulse	$\delta(t)$ unit impulse	1	1
unit step	$u(t)$, $\gamma(t)$ *(Note)*	$\Gamma(s)=\frac{1}{s}$	$\frac{z}{z-1}$
unit step time delay	$u(t-\tau)$	$\frac{1}{s}e^{-\tau s}$	
ramp	t	$\frac{1}{s^2}$	$T\frac{z}{(z-1)^2}$
parabola	t^2	$\frac{2}{s^3}$	$T^2\frac{z(z+1)}{(z-1)^3}$
t^n (n is integer)	t^n	$\frac{n!}{s^{(n+1)}}$	
exponential	e^{-at}	$\frac{1}{s+a}$	$\frac{z}{z-e^{-aT}}$
power	b^k $(b=e^{-aT})$		$\frac{z}{z-b}$
time multiplied exponential	te^{-at}	$\frac{1}{(s+a)^2}$	$T\frac{ze^{-aT}}{(z-e^{-aT})^2}$
Asymptotic exponential	$\frac{1}{a}(1-e^{-at})$	$\frac{1}{s(s+a)}$	$\frac{z(1-e^{-aT})}{a(z-1)(z-e^{-aT})}$
double exponential	$\frac{e^{-at}-e^{-bt}}{(b-a)}$	$\frac{1}{(s+a)(s+b)}$	$\frac{z(e^{-aT}-e^{-bT})}{(b-a)(z-e^{-aT})(z-e^{-bT})}$
asymptotic double exponential	$\frac{1}{ab}\left(1-\frac{be^{-at}-ae^{-bt}}{(b-a)}\right)$	$\frac{1}{s(s+a)(s+b)}$	
asymptotic critically damped	$\frac{1}{a^2}(1-e^{-at}-ate^{-at})$	$\frac{1}{s(s+a)^2}$	$\frac{(1-e^{-Ta}(1+Ta))z^2+e^{-Ta}(Ta-1+e^{-Ta})z}{a^2(z-e^{-Ta})^2(z-1)}$
differentiated critically damped	$(1-at)e^{-at}$	$\frac{s}{(s+a)^2}$	$\frac{z(z-(Ta+1)e^{-Ta})}{(z-e^{-Ta})^2}$
sine	$\sin(\omega_0 t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\frac{z\sin(\omega_0 T)}{z^2-2z\cos(\omega_0 T)+1}$
cosine	$\cos(\omega_0 t)$	$\frac{s}{s^2+\omega_0^2}$	$\frac{z(z-\cos(\omega_0 T))}{z^2-2z\cos(\omega_0 T)+1}$
decaying sine	$e^{-at}\sin(\omega_d t)$	$\frac{\omega_d}{(s+a)^2+\omega_d^2}$	$\frac{ze^{-aT}\sin(\omega_d T)}{z^2-2ze^{-aT}\cos(\omega_d T)+e^{-2aT}}$
decaying cosine	$e^{-at}\cos(\omega_d t)$	$\frac{s+a}{(s+a)^2+\omega_d^2}$	$\frac{z(z-e^{-aT}\cos(\omega_d T))}{z^2-2ze^{-aT}\cos(\omega_d T)+e^{-2aT}}$
generic decaying oscillatory	$e^{-at}\left(B\cos(\omega_d t)+\frac{C-aB}{\omega_d}\sin(\omega_d t)\right)$	$\frac{Bs+C}{(s+a)^2+\omega_d^2}$	
generic decaying oscillatory (alternate)	$Me^{-at}\cos(\omega_d t+\phi)$ $M=\sqrt{B^2+\left(\frac{C-aB}{\omega_d}\right)^2}$ $\phi=-\mathrm{atan}\left(\frac{C-aB}{B\omega_d}\right)=-\mathrm{atan2}(C-aB,B\omega_d)$ *(Note)*	$\frac{Bs+C}{(s+a)^2+\omega_d^2}$	
Z-domain generic decaying oscillatory	$\sqrt{\frac{a^2d^2+b^2-2abc}{d^2-c^2}}\cdot d^n\cdot\cos(\omega_d n+\phi)$ $\omega_d=\mathrm{acos}\left(-\frac{c}{d}\right)$, $\phi=\mathrm{atan}\left(\frac{ac-b}{a\sqrt{d^2-c^2}}\right)$ *(Note)*		$\frac{az^2+bz}{z^2+2cz+d^2}$, $d>0$
Prototype Second Order System ($\zeta<1$, underdampded)			
Prototype 2nd order lowpass step response	$1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_0 t}\sin\left(\omega_0\sqrt{1-\zeta^2}t+\phi\right)$ $\phi=\mathrm{acos}(\zeta)$	$\frac{\omega_0^2}{s(s^2+2\zeta\omega_0 s+\omega_0^2)}$	$\frac{z}{z-1}-\frac{1}{\sqrt{1-\zeta^2}}\frac{z^2\sqrt{1-\zeta^2}+ze^{-\zeta\omega_0 T}\sin\left(\omega_0\sqrt{1-\zeta^2}T-\phi\right)}{z^2-2ze^{-\zeta\omega_0 T}\cos\left(\omega_0\sqrt{1-\zeta^2}T\right)+e^{-2\zeta\omega_0 T}}$ $\phi=\mathrm{acos}(\zeta)$
Prototype 2nd order lowpass impulse response	$\frac{\omega_0}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_0 t}\sin\left(\omega_0\sqrt{1-\zeta^2}t\right)$	$\frac{\omega_0^2}{s^2+2\zeta\omega_0 s+\omega_0^2}$	$\frac{\omega_0}{\sqrt{1-\zeta^2}}\frac{ze^{-\zeta\omega_0 T}\sin\left(\omega_0\sqrt{1-\zeta^2}T\right)}{z^2-2ze^{-\zeta\omega_0 T}\cos\left(\omega_0\sqrt{1-\zeta^2}T\right)+e^{-2\zeta\omega_0 T}}$
Prototype 2nd order bandpass impulse response	$-\frac{2\zeta\omega_0}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_0 t}\sin\left(\sqrt{1-\zeta^2}\cdot\omega_0 t-\phi\right)$ $\phi=\mathrm{acos}(\zeta)$	$\frac{2\zeta\omega_0 s}{s^2+2\zeta\omega_0 s+\omega_0^2}$	$-\frac{2\zeta\omega_0 T}{\sqrt{1-\zeta^2}}\frac{-z^2\sqrt{1-\zeta^2}+ze^{-\zeta\omega_0 T}\sin\left(\sqrt{1-\zeta^2}\cdot\omega_0 T+\phi\right)}{z^2-2ze^{-\zeta\omega_0 T}\cos\left(\sqrt{1-\zeta^2}\cdot\omega_0 T\right)+e^{-2\zeta\omega_0 T}}$ $\phi=\mathrm{acos}(\zeta)$

Entry	Time Domain *(Note)*	Laplace Domain	Z
unit impulse	$\delta(t)$ unit impulse	1	
unit step	$u(t), \quad \gamma(t)$ *(Note)*	$\Gamma(s) = \frac{1}{s}$	
unit step time delay	$u(t-\tau)$	$\frac{1}{s}e^{-\tau s}$	
ramp	t	$\frac{1}{s^2}$	
parabola	t^2	$\frac{2}{s^3}$	
t^n (n is integer)	t^n	$\frac{n!}{s^{(n+1)}}$	
exponential	e^{-at}	$\frac{1}{s+a}$	
power	$b^k \qquad (b = e^{-aT})$		
time multiplied exponential	te^{-at}	$\frac{1}{(s+a)^2}$	
Asymptotic exponential	$\frac{1}{a}(1-e^{-at})$	$\frac{1}{s(s+a)}$	
double exponential	$\frac{e^{-at}-e^{-bt}}{(b-a)}$	$\frac{1}{(s+a)(s+b)}$	
asymptotic double exponential	$\frac{1}{ab}\left(1-\frac{be^{-at}-ae^{-bt}}{(b-a)}\right)$	$\frac{1}{s(s+a)(s+b)}$	
asymptotic critically damped	$\frac{1}{a^2}(1-e^{-at}-ate^{-at})$	$\frac{1}{s(s+a)^2}$	
differentiated critically damped	$(1-at)e^{-at}$	$\frac{s}{(s+a)^2}$	
sine	$\sin(\omega_0 t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	
cosine	$\cos(\omega_0 t)$	$\frac{s}{s^2+\omega_0^2}$	
decaying		ω_d	